

# THE BIRKHOFF THEOREM IN THE QUANTUM THEORY OF TWO-DIMENSIONAL DILATON GRAVITY

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## ABSTRACT

In classical two-dimensional pure dilaton gravity, and in particular in spherically symmetric pure gravity in  $d$  dimensions, the generalized Birkhoff theorem states that, for a suitable choice of coordinates, the metric coefficients are only functions of a single coordinate. It is interesting to see how this result is recovered in quantum theory by the explicit construction of the Hilbert space. We examine the CGHS model, enforce the set of auxiliary conditions that select physical states à la Gupta-Bleuler, and prove that the matrix elements of the metric and of the dilaton field obey the classical requirement. We introduce the mass operator and show that its eigenvalue is the only gauge invariant label of states. Thus the Hilbert space is equivalent to that obtained by quantum mechanical treatment of the static case. This is the quantum form of the Birkhoff theorem for this model.

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## 1. Introduction.

The quantization of two-dimensional dilaton theories has received much attention, because of its connection to string theory and also, for some choices of the potential, to spherically symmetric pure gravitational  $d$ -dimensional configurations [1-11].

Classically, the two-dimensional pure dilaton theories obey the generalized Birkhoff theorem, namely their solutions depend, in suitable coordinates, on a single variable; we shall refer to this property as “staticity”. This is a well known property (see e.g. [12,13]). The famous case is gravity in four dimensions, where, by a suitable choice of coordinates, a purely gravitational spherically symmetric configuration can be cast in the form

$$ds^2 = -A(r)dt^2 + N(r)dr^2 + r^2 d\Omega^2, \quad (1)$$

where  $d\Omega^2$  is the metric of the unit two-sphere and  $(r, t)$  are coordinates defined on  $\mathbb{R}^+$  and  $\mathbb{R}$  respectively. The coefficients of the metric are only functions of the radial coordinate  $r$ : this is the content of the classical Birkhoff theorem. Now it is interesting to see how this property appears in the quantum case, where it reduces the two-dimensional field theory to quantum mechanics.

In the quantum framework there are two possible approaches. The two-dimensional theory can be quantized as a field theory, and then reduction enforced, or one can quantize the system obtained after the classical static property has been introduced (direct static quantization), thus imposing staticity from the beginning [14,15]. In the latter case one is confronted with a constrained quantum mechanical system that for the Schwarzschild case has been explicitly solved, and its Hilbert space determined [14].

It is then interesting to study in detail the reduction of the field theory to quantum mechanics and its connection to the direct static quantization. In this paper we discuss the CGHS model [4]. We linearize and implement the constraints à la Gupta-Bleuler. This scheme of quantization allows the algebraic construction of physical states. In this scheme, the result is equivalent to the quantization of pure scalar-longitudinal electrodynamics (apart from the presence of the mass operator). In this way the staticity property appears explicitly. The expectation values of the metric and dilaton fields give back the corresponding classical static formulae. The choice of the quantum state corresponds to the classical choice of coordinates.

The only relevant physical fact is the existence of the mass operator, which is the zero mode of the dilaton field and commutes with the scalar and longitudinal modes of the D'Alembert fields. The vacuum is thus characterized by the mass quantum number. The field theory is reduced to quantum mechanics.

We may conjecture that the same line should be followed in the more complicated case of general spherically symmetric pure gravity in  $d$  dimensions, and in particular in the Schwarzschild case, leading to the quantum mechanics formulated in [14].

## 2. Action and Hamiltonian Formalism.

Our starting point is the two-dimensional action

$$S = \int d^2x \sqrt{-g} \left[ \varphi R - \frac{\lambda}{2} \right], \quad (2)$$

where  $g_{\mu\nu}$  is a two-dimensional metric and  $\varphi$  is the “dilaton field” (for  $R$  we follow the conventions of [16]). This model is related by a Weyl transformation to the CGHS [4] model. As in [9] we write the two-dimensional metric as

$$g_{\mu\nu} = \rho \begin{pmatrix} \alpha^2 - \beta^2 & \beta \\ \beta & -1 \end{pmatrix}. \quad (3)$$

Here  $\alpha(x_0, x_1)$  and  $\beta(x_0, x_1)$  play the role of the lapse function and of the shift vector respectively;  $\rho(x_0, x_1)$  represents the dynamical gravitational degree of freedom.

It is convenient to introduce the variable  $f = \ln \rho$ . Using the ansatz (3) the action (2) can be written in the Hamiltonian form (see [9]) as

$$S = \int d^2x \left[ \dot{f} \pi_f + \dot{\varphi} \pi_\varphi - \alpha \mathcal{H} - \beta \mathcal{P} \right], \quad (4)$$

where  $\pi_f$  and  $\pi_\varphi$  are the conjugate momenta to  $f$  and  $\varphi$  respectively, and  $\mathcal{H}$  and  $\mathcal{P}$  are the super-Hamiltonian and the super-momentum:

$$\mathcal{H} = \pi_f \pi_\varphi + f' \varphi' - 2\varphi'' + \frac{\lambda}{2} e^f, \quad (5a)$$

$$\mathcal{P} = 2\pi_f' - \pi_\varphi \varphi' - \pi_f f'. \quad (5b)$$

The Hamiltonian equations of motion are

$$\dot{\varphi} = \alpha \pi_f - \beta \varphi', \quad \dot{f} = \alpha \pi_\varphi - \beta f' - 2\beta', \quad (6a)$$

$$\dot{\pi}_\varphi = \frac{\partial}{\partial x_1} (\alpha f' + 2\alpha' - \beta \pi_\varphi), \quad \dot{\pi}_f = -\frac{\lambda}{2} \alpha e^f + \frac{\partial}{\partial x_1} (\alpha \varphi' - \beta \pi_f), \quad (6b)$$

and obviously the constraints

$$\mathcal{H} = 0, \quad \mathcal{P} = 0. \quad (7)$$

We may also define a functional  $M$  [7, 12, 13] of  $f$  and  $\varphi$  conserved under time and space translations (analogous to the Schwarzschild mass). In our notations  $M$  is given by

$$M = \frac{\lambda}{2} \varphi + e^{-f} (\pi_f^2 - \varphi'^2). \quad (8)$$

It is straightforward to prove that  $\dot{M} = M' = 0$  using the equations of motion and the constraints.

The Birkhoff classical staticity can be stated as follows. We may set  $\alpha = 1$  and  $\beta = 0$  and introduce the coordinates

$$u = \frac{1}{2}(x_0 + x_1), \quad v = \frac{1}{2}(x_0 - x_1); \quad (9)$$

the two-dimensional line element corresponding to the metric tensor (3) becomes

$$ds^2 = 4\rho(u, v)dudv. \quad (10)$$

A metric of this form is static if and only if [12]  $\rho$  can be cast in the form

$$\rho(u, v) = h(\Psi) \frac{da(u)}{du} \frac{db(v)}{dv}, \quad \Psi \equiv a(u) + b(v), \quad (11)$$

where  $a$  and  $b$  are two suitable functions. This will be useful later.

In the variables (9) the equations of motion and constraints read [12]

$$\partial_u \partial_v f = 0, \quad \partial_u \partial_v \varphi + \frac{\lambda}{2} e^f = 0, \quad (12a)$$

$$\partial_u (e^{-f} \partial_u \varphi) = \partial_v (e^{-f} \partial_v \varphi) = 0, \quad (12b)$$

where (12b) coincides with  $\mathcal{H} \pm \mathcal{P} = 0$ . The solution of these equations is static:

$$\rho \equiv e^f = \frac{dF}{d\Psi} \partial_u \Psi \partial_v \Psi, \quad (13a)$$

$$\lambda \varphi = F(\Psi) = C_0 e^{-\lambda \Psi/2} + 2M, \quad (13b)$$

$$\partial_u \partial_v \Psi = 0. \quad (13c)$$

One sees that  $M$  appears as a zero mode of the field  $\varphi$ . The freedom of choosing  $\Psi$  as a solution of (13c) is related to the reparametrizations of Eq. (11).

Now let us recall the canonical free field formalism that will be the starting point for the quantum theory.

### 3. Canonical Transformation to free fields.

Let us use the transformation [9]

$$\begin{aligned} A_0 &= \frac{2}{\lambda} e^{-f/2} (\pi_f \cosh \Sigma - \varphi' \sinh \Sigma), \quad \pi^0 = -\lambda e^{f/2} \cosh \Sigma - \lambda A'_1, \\ A_1 &= \frac{2}{\lambda} e^{-f/2} (\pi_f \sinh \Sigma - \varphi' \cosh \Sigma), \quad \pi^1 = \lambda e^{f/2} \sinh \Sigma + \lambda A'_0, \end{aligned} \quad (14)$$

where

$$\Sigma = \frac{1}{2} \int^x dx' \pi_\varphi(x'). \quad (15)$$

The above transformation is canonical. Using (14) the two constraints become

$$\mathcal{H} = \frac{1}{2\lambda} \pi^\alpha \pi_\alpha + \frac{\lambda}{2} A'^\alpha A'_\alpha = 0, \quad (16a)$$

$$\mathcal{P} = -\pi^\alpha A'_\alpha = 0. \quad (16b)$$

We introduce the metric and the Levi Civita tensors as

$$\eta^{\alpha\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad \varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (17)$$

The functional  $M$  defined in (8) is represented as

$$M = M_0 + \frac{\lambda}{4} \int_b^x dx' (\varepsilon^{\alpha\beta} \pi_\alpha A_\beta + \lambda A^\alpha A'_\alpha). \quad (18)$$

Here  $M_0$  must be a constant since  $M$  is independent of  $x_0$ ; it corresponds to a constant in  $\varphi$  (zero mode). On the equations of motion we have  $M = M_0$ .  $\dot{M}$  and  $M'$  are proportional to  $\mathcal{C}^\alpha$  defined in next (19).

The constraints (16) can be cast in a simpler form. Indeed, Eqs. (16) are equivalent to the linearized constraints

$$\mathcal{C}^\alpha \equiv \pi^\alpha - \lambda \varepsilon^{\alpha\beta} A'_\beta = 0. \quad (19)$$

The sign ambiguity arising in the process of linearization is removed using the canonical transformation (14). Finally the general system described by the action (2) is equivalent to the system described by the Hamiltonian density

$$H = \mathcal{H} + l_\alpha \mathcal{C}^\alpha, \quad (20)$$

where  $l_\alpha$  are two Lagrange multipliers that imply  $\mathcal{C}^\alpha = 0$  and thus  $\mathcal{H} = 0$  and  $\mathcal{P} = 0$ .

Let us discuss how staticity can be recovered from (20). The equations of motion are

$$\lambda \dot{A}_\alpha = \eta_{\alpha\beta} \pi^\beta, \quad \dot{\pi}^\alpha = \lambda \eta^{\alpha\beta} A''_\beta \quad (21)$$

and the solution is

$$A_\alpha = U_\alpha(u) + V_\alpha(v). \quad (22)$$

Now let us implement the constraints (19), that correspond to

$$\eta^{\alpha\beta} \partial_\alpha A_\beta = 0, \quad \varepsilon^{\alpha\beta} \partial_\alpha A_\beta = 0, \quad \text{or,} \quad U_0(u) = U_1(u), \quad V_0(v) = -V_1(v). \quad (23)$$

It is easy to prove that the above solution is static and coincides with (13). To this aim we need to obtain  $f$  and  $\varphi$  as functions of  $A_\alpha$  inverting (14):

$$\rho \equiv e^f = \frac{2}{\lambda} \mathcal{H} - 2A'^\alpha A'_\alpha + \frac{2}{\lambda} \varepsilon^{\alpha\beta} \pi_\alpha A'_\beta, \quad \varphi = \frac{2M}{\lambda} - \frac{\lambda}{2} A^\alpha A_\alpha. \quad (24)$$

Substituting Eqs. (22,23) in (24), and using  $\mathcal{H} = 0$ , one obtains

$$\rho \equiv e^f = 4\lambda \frac{dU_0(u)}{du} \frac{dV_0(v)}{dv}, \quad (25a)$$

$$\varphi = \frac{2M}{\lambda} - 2\lambda U_0(u) V_0(v). \quad (25b)$$

The above solution coincides with (13), with

$$C_0 e^{-\lambda\Psi/2} = -2\lambda^2 U_0(u) V_0(v). \quad (26)$$

The property of staticity of the classical solution is thus represented by the conditions (23), to be implemented in the canonical quantization.

#### 4. Quantization.

The quantization starts from the introduction of the Lagrangian<sup>1</sup>

$$\mathcal{L} = \frac{1}{2} \partial_\mu A_\alpha \partial_\nu A_\beta \eta^{\mu\nu} \eta^{\alpha\beta}. \quad (27)$$

The commutation relations are

$$[A_\alpha(x), A_\beta(y)] = -\eta_{\alpha\beta} \int \frac{d^2k}{2\pi} \delta(k^2) \varepsilon(k_0) e^{ik(x-y)}. \quad (28)$$

The field expansion is

$$A_\alpha = \int_{-\infty}^{\infty} \frac{dk}{2\sqrt{\pi\omega}} \{b_\alpha(k) e^{-i\omega x_0 + ikx_1} + b_\alpha^\dagger(k) e^{i\omega x_0 - ikx_1}\}, \quad (29)$$

where  $\omega = |k|$ . From (22) we obtain

$$\begin{aligned} U_\alpha &= \int_0^\infty \frac{dk}{2\sqrt{\pi k}} \{a_\alpha(k) e^{-2iku} + a_\alpha^\dagger(k) e^{2iku}\}, \\ V_\alpha &= \int_0^\infty \frac{dk}{2\sqrt{\pi k}} \{b_\alpha(k) e^{-2ikv} + b_\alpha^\dagger(k) e^{2ikv}\}, \end{aligned} \quad (30)$$

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<sup>1</sup> From now on we set  $\lambda=1$ . There is no real loss of generality while the formulae become more elegant.

and  $a_\alpha(k) = b_\alpha(-k)$ ,  $k > 0$ . Consequently ( $k > 0$ ) the non-vanishing commutators are

$$[a_\alpha(k), a_\beta^\dagger(k')] = \eta_{\alpha\beta} \delta(k - k'), \quad [b_\alpha(k), b_\beta^\dagger(k')] = \eta_{\alpha\beta} \delta(k - k'). \quad (31)$$

It follows that

$$[U_\alpha(u_1), U_\beta(u_2)] = -\frac{i}{4} \eta_{\alpha\beta} \varepsilon(u_1 - u_2), \quad [V_\alpha(v_1), V_\beta(v_2)] = -\frac{i}{4} \eta_{\alpha\beta} \varepsilon(v_1 - v_2). \quad (32)$$

The operator  $M_0$  that is the zero mode of  $\varphi$  commutes with all the annihilation and creation operators since by Eqs. (14) it is not contained in the fields  $A_\alpha$ . This will be very important later.

The classical constraints (19,23) cannot be implemented operatorially, since as operator equations they are in contrast with the quantization rules (28-32). A way of solving for the Hilbert space is to introduce an indefinite metric in the space of states as in the Gupta-Bleuler procedure [17]. Following the usual lore we proceed defining the vacuum as

$$a_\alpha(k)|0\rangle = 0, \quad b_\alpha(k)|0\rangle = 0. \quad (33)$$

This leads to negative norm states. Now we implement the constraints by requiring that, for each oscillation mode, physical states be selected by

$$(a_0(k) - a_1(k))|\Psi\rangle = 0, \quad (b_0(k) + b_1(k))|\Psi\rangle = 0. \quad (34)$$

Let us introduce

$$q_a = a_0 - a_1, \quad q_b = b_0 + b_1, \quad [q_{a,b}(k), q_{a,b}^\dagger(k')] = 0, \quad (35)$$

and the states  $|\{n_a, n_b\}\rangle$  defined as

$$|\{n_a, n_b\}\rangle \equiv q_a^\dagger(k_1) \dots q_a^\dagger(k_{n_a}) q_b^\dagger(k'_1) \dots q_b^\dagger(k'_{n_b}) |0\rangle. \quad (36)$$

These states have zero norm if  $n_a \neq 0$  or  $n_b \neq 0$ . The general solution of the constraints is then

$$|\Psi\rangle = \sum_{n_a, n_b} \int d^{n_a} k \int d^{n_b} k' C_{n_a n_b}(k_1, \dots, k_{n_a}; k'_1, \dots, k'_{n_b}) |\{n_a, n_b\}\rangle. \quad (37)$$

The norm of this state is

$$\langle \Psi | \Psi \rangle = |C_{00}|^2. \quad (38)$$

One can check that the classical constraints hold for expectation values. After some algebra one finds

$$\langle \Psi | : \mathcal{H} : | \Psi \rangle = 0, \quad \langle \Psi | : \mathcal{P} : | \Psi \rangle = 0, \quad (39)$$

where the normal ordering with annihilation operators on the right must be used. It is easy to check that all matrix elements of  $\mathcal{H}$  and  $\mathcal{P}$  among physical states vanish.

Analogously one can calculate the matrix elements of  $\rho$ , of the mass  $M$ , and of the scalar field  $\varphi$ . Using (24) and (37) the expectation value of  $\rho$  is

$$\langle \Psi | : \rho(u, v) : | \Psi \rangle = 4 \frac{dF(u)}{du} \frac{dG(v)}{dv}, \quad (40)$$

where

$$\begin{aligned} F(u) &= \int \frac{dk}{2\sqrt{\pi k}} (C_{00}^* C_{10}(k) e^{-2iku} + C_{00} C_{10}(k)^* e^{2iku}), \\ G(v) &= \int \frac{dk}{2\sqrt{\pi k}} (C_{00}^* C_{01}(k) e^{-2ikv} + C_{00} C_{01}(k)^* e^{2ikv}). \end{aligned} \quad (41)$$

The result (40) is analogous to the classical relation (25a); we have of course

$$F(u)G(v) = \langle \Psi | U_0(u) V_0(v) | \Psi \rangle. \quad (42)$$

Note that  $\langle \Psi | : \rho(u, v) : | \Psi \rangle$  has the form

$$\langle \Psi | : \rho(u, v) : | \Psi \rangle = h(a(u) + b(v)) \frac{da(u)}{du} \frac{db(v)}{dv}, \quad (43)$$

which is the essence of classical staticity.

Let us now consider the operator  $M$ , Eq. (18). The quantity  $I$  that is the integrand in (18) classically vanishes. In the quantum case, each term in  $I$  contains one of the operators  $q_{a,b}$  or  $q_{a,b}^\dagger$ . Adopting a normal ordering, the matrix elements of  $I$  between physical states vanish. This corresponds to the classical property. So,

$$\langle \Psi_2 | M | \Psi_1 \rangle = \langle \Psi_2 | M_0 | \Psi_1 \rangle. \quad (44)$$

Further, the operator  $M_0$  commutes with the creation and annihilation operators of  $A_\alpha$ , since  $M_0$  is the zero mode of the field  $\varphi$ . So we must characterize the vacuum by a further quantum number:

$$M_0 |0; m\rangle = m |0; m\rangle. \quad (45)$$

Eq. (45) is of the utmost interest. There are infinite vacua, differing by the eigenvalue of  $M_0$ . The only gauge invariant label of a state is  $m$ . This result is similar to the case of the Schwarzschild metric discussed in [14], where staticity was imposed from the beginning, reducing the problem to quantum mechanics, and states were labeled by the eigenvalues of the mass operator.

Finally, the expectation value of  $\varphi$  reads

$$\begin{aligned} \langle \Psi; m | : \varphi(u, v) : | \Psi; m \rangle &= 2m - 2 \langle \Psi; m | U_0(u) V_0(v) | \Psi; m \rangle \\ &= 2m - 2F(u)G(v). \end{aligned} \quad (46)$$



in analogy to (25b).

We conclude with two interesting remarks. The first is that the roles of  $A_0$  and  $A_1$  can be interchanged, i.e. the sign in Eq. (28) can be changed, because the condition (16a) shows that the choice of the “right” metric field is irrelevant. The operators  $q_a$  and  $q_b$  will again contain one operator with wrong metric and one with right metric; nothing changes in the construction (37) of the physical states.

Second remark: our quantization rule (28) amounts to assuming that  $x_0$  is time, namely that the canonical equal  $x_0$  commutators for  $A_0$  hold. The determination of the physical states is actually independent of which coordinate is chosen as time. Indeed, let us suppose  $x_1$  to be the timelike variable, and proceed by canonical  $x_1$  quantization for  $A_0$ . In that case the rule (28) is suitably modified. In the rules (31) the commutators of the  $b_\alpha$  change sign: now  $b_0$  has wrong metric while  $b_1$  has the correct one. Again, in  $q_b$  there appears one operator with the right and one with the wrong metric and the construction of physical states, Eq. (36), remains unchanged.

## 5. Conclusions.

Classically, taking into account the constraints (23), all the field theory tells us is just that there is a single free field whose degrees of freedom correspond to reparametrization of the static coordinate, Indeed, a choice for  $U_0(u)$ ,  $V_0(v)$  defines  $\Psi$ , and the different choices correspond to different solutions of (13c). In the quantum theory, the physics contained in  $A_\alpha$  is pure gauge, equivalent to free electrodynamics of longitudinal and scalar photons, and in this respect the state  $|\Psi\rangle$  conveys the information correspondent to the classical case, see (40) and (46).

What is physically important is the eigenvalue of the constant operator  $M_0$ , Eq. (45). The vacuum has a quantum number: the eigenvalue of the mass operator. Thus the theory is reduced essentially to quantum mechanics, while the rest is coordinate reparametrization. One may conjecture that this mechanism is at the basis of the dimensional reduction for all the quantum field models for which classically the Birkhoff theorem holds. When a general potential appears in (3) the problem is the existence and identification of the canonical transformation, analogous to (14), that leads to free fields. If it is so, the quantum mechanics [14] that is obtained quantizing the spherically symmetric metric (1) in its static form contains the same physical information as the quantum field theory.

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